REMARKS

In response to the Office Action mailed August 16, 2007, Applicant respectfully requests reconsideration. Claims 1-2 and 5-18 were previously pending in this application. By this amendment, claims 1-2 and 18 have been amended. As a result, claims 1-2 and 5-18 are pending for examination with claims 1 and 18 being independent claims. No new matter has been added. The application as presented is believed to be in condition for allowance.

Initially, Applicant's representatives thank Examiner Pérez-Gutiérrez and Examiner Ajibade-Akonai for the courtesies extended during a telephone interview conducted on November 7, 2007, in which Scott J. Gerwin (Reg. No. 57,866) and Jeffrey C. O'Neill participated. The substance of this interview is summarized herein.

Rejections Under 35 U.S.C. § 103

Claims 1-2 and 5-18 are rejected under 35 U.S.C. 103(a) as purportedly being unpatentable over Chow, U.S. Patent No. 6,771,966 (hereinafter "Chow") in view of one or more other references. Claim 1-2, 5, 7-9, and 13-18 are rejected as being unpatentable over Chow in view of Sengoku et al. "Cellular Mobile Communication Systems and a Channel Assignment Using Neural Networks" (hereinafter "Sengoku"). Claim 6 is rejected as being unpatentable over Chow in view of Sengoku and further in view of Csapo, U.S. Patent Application 20030202497 (hereinafter "Csapo"). Claims 10 and 11 are rejected as being unpatentable over Chow in view of Sengoku and further in view of Hung et al., U.S. Patent Application 20050058111 (hereinafter "Hung"). Claim 12 is rejected as being unpatentable over Chow in view of Sengoku and further in view of Stanley, U.S. Patent No. 6,836,467 (hereinafter "Stanley"). Each of these rejections is respectfully traversed.

Discussion of Chow

Chow discloses presenting information about a wireless network in a graph. In FIGS. 3-4, 6-9, 11-13, and 16-17, Chow shows graphs of wireless networks. In each of these graphs, the vertices (*i.e.*, the dots where lines meet or intersect) of the graphs represent communications nodes. (Col. 12, lines 44-48). In the graphs, edges (*i.e.*, lines) drawn between the vertices indicate communications links between the corresponding communications nodes. For example,

in FIG. 3A, the edges indicate potential communications links between the nodes, and each vertex is connected to every other vertex by an edge. (Col. 12, lines 48-58). In FIGS. 3B and 3C, the edges indicate communications links that do not interfere with each other. (Col. 14, lines 20-47).

Other graphs in Chow present more information about the communications network. For example, in FIGS. 4A-4D, each vertex (*i.e.*, each dot) represents a communications node, and each communications node is specifically marked as a transmitter or a receiver. (Col. 14, line 55 – col. 15, line 5). Each transmitter in FIGS. 4A-4D has an intended receiver. (Id.). The communications link from a transmitter to an intended receiver is denoted by an edge (*i.e.*, a line) marked as a "signal path," and the communications link from a transmitter to an unintended receiver is denoted by an edge marked as an "interference path." (Id.).

In each of the graphs presented by Chow in FIGS. 3-4, 6-9, 11-13, and 16-17, a vertex in the graph represents a communications node, and an edge between two vertices in a graph represents a communications link between the corresponding communications nodes.

Claims 1-2 and 5-18

Amended claims 1 and 18 recite a method and a computer-readable medium, respectively, for "modeling wireless interference among wireless links between a plurality of wireless nodes in a wireless network." The claims comprise "creating a graph from the connectivity information, wherein each identified wireless link is represented as a vertex in the graph" and "an edge is created between a first vertex and a second vertex in the graph if the corresponding wireless links interfere with one another."

During the telephone interview, Applicant's representatives explained that Chow does not teach or disclose modeling a wireless network where the wireless links are represented by vertices in the graph and the edges between the vertices of the graph indicate whether the corresponding wireless links interfere with one another. Applicant's representatives explained that Chow discloses using vertices to represent only radio nodes and Chow does not disclose using vertices to represent wireless links. Further, Chow discloses using edges between vertices to represent a wireless link between the radio nodes corresponding to the connected vertices, and Chow does not disclose using edges to represent interference between two wireless links.

Additionally, Applicant's representatives explained that one of skill in the art, would not interpret the lines labeled "signal path" in FIG. 4A of Chow to be a "vertex" in a graph, as that term is used in the claim. Applicant's representatives cited to the Examiners a textbook on graph theory that defines vertices and edges as follows: "[G]raphs are drawn with dots and lines. . . . The dots . . . are called **vertices** and the lines that connect the vertices are called **edges**." Richard Johnsonbaugh, *Discrete Mathematics*, p. 305 (4th Ed. 1997). Applicant's representatives also cited to the Examiners a technical dictionary that defines a vertex as "a member of the set of points that are connected by the edges." *McGraw-Hill Dict. of Sci. and Tech. Terms* (6th Ed. 2003). Copies of the cited portions of these references are appended hereto.

Applicant's representatives further explained that these definitions are consistent with the way that these terms are used in the specification, pointing out that FIG. 4 of the present application shows a graph comprising two circles connected by a line. (¶ 41). The circles are indicated by reference numbers 300 and 400, and the line is indicated by reference number 402.[1] The specification states that the two circles are two vertices in the graph and the line is an edge between the two vertices. (¶ 41). The terms "vertex" and "edge" are thus used in the specification in accordance with their plain and customary meaning. Thus, these terms should be construed in accordance with these meanings.

During the interview, Applicant's representatives presented proposed claim amendments to the Examiners to clarify that the vertices and edges recited in the claims are included in a graph. The Examiners stated, based on the above-discussed references defining the terms "vertex" and "edge" in the context of a graph, that they agreed that interpreting the lines labeled "signal path" in FIG. 4A to be vertices is improper. While understandably reserving final judgment until receiving a formal response, the Examiners indicated that if the claims were amended in the manner proposed, such that the wireless links were represented in a graph, the amended claims would distinguish the prior art of record.

As discussed above, Chow does not teach or suggest creating a graph wherein "each identified wireless link is represented as a vertex in the graph" and wherein "an edge is created

[1] Note that in the Amendment dated June 21, 2006, applicants corrected a typographical error in FIG. 4. FIG. 4 previously referred to the two vertices in FIG. 4 by reference numbers 300 and 312, but paragraph 41 of the specification used reference numbers 300 and 400 to refer to the vertices. In the Amendment of June 21, 2006, Applicants corrected this typographical error by replacing reference number 312 in FIG. 4 with reference number 400.

between a first vertex and a second vertex in the graph if the corresponding wireless links interfere with one another." Rather, in FIG. 4A, Chow shows a graph in which communications nodes are represented as vertices in a graph and wireless links between communications nodes are represented as edges between the corresponding vertices.

Sengoku does not cure the infirmities of Chow. Without acceding to the propriety of the asserted combination, Applicant respectfully points out that the prior art of record fails to satisfy all of the limitations recited by claims 1 and 18.

As should be appreciated from the foregoing, claims 1 and 18 patentably distinguish over Chow in combination with Sengoku. Accordingly, it is respectfully requested that the rejection of claims 1 and 18 under 35 U.S.C. § 103 be withdrawn.

Claims 2 and 5-17 depend from claim 1 and are patentable for at least the same reasons. Accordingly, it is respectfully requested that the rejections of claims 2 and 5-17 be withdrawn.

35 U.S.C. § 101

As previously presented, claim 18 recited a "computer-readable medium containing computer-executable instructions for modeling wireless interference among wireless links between a plurality of wireless nodes in a wireless network." During the interview, the Examiners, while noting that the Office Action does not set forth any §101 rejection, questioned whether claim 18 is directed to statutory subject matter. Specifically, the Examiners noted that, in ¶ 30 on page 8 of the specification, the term "computer-readable medium" is defined to include "both storage media and communications media" and that "communications media" includes a "modulated signal." The Examiners questioned whether a claim that reads on a signal is statutory.

Applicants believe that claim 18 as previously presented was statutory. However, to allay any concerns that the Examiners may have, Applicant has amended claim 18 to recite a "computer-readable storage medium" rather than just a "computer-readable medium." Thus, it should be clear that claim 18 is directed only to computer-readable storage media and not to communications media.

CONCLUSION

A Notice of Allowance is respectfully requested. The Examiner is requested to call the undersigned at the telephone number listed below if this communication does not place the case in condition for allowance.

If this response is not considered timely filed and if a request for an extension of time is otherwise absent, Applicant hereby requests any necessary extension of time. If there is a fee occasioned by this response, including an extension fee, that is not covered by an enclosed check, please charge any deficiency to Deposit Account No. 23/2825.

Dated: November 16, 2007

Respectfully submitted,

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McGRAW-HILL DICTIONARY OF SCIENTIFIC AND TECHNICAL TERMS

Sixth Edition

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On the cover: Representation of a fullerene molecule with a noble gas atom trapped inside. At the Permian-Triassic sedimentary boundary the noble gases helium and argon have been found trapped inside fullerenes. They exhibit isotope ratios quite similar to those found in meteorites, suggesting that a fireball meteorite or asteroid exploded when it hit the Earth, causing major changes in the environment. *(Image copyright © Dr. Luann Becker. Reproduced with permission.)*

Over the six editions of the Dictionary, material has been drawn from the following references: G. M. Garrity et al., *Taxonomic Outline of the Procaryotes,* Release 2, Springer-Verlag, January 2002; D. W. Linzey, *Vertebrate Biology,* McGraw-Hill, 2001; J. A. Pechenik, *Biology of the Invertebrates,* 4th ed., McGraw-Hill, 2000; *U.S. Air Force Glossary of Standardized Terms,* AF Manual 11-1, vol. 1, 1972; F. Casey, ed., *Compilation of Terms in Information Sciences Technology,* Federal Council for Science and Technology, 1970; *Communications-Electronics Terminology,* AF Manual 11-1, vol. 3, 1970; P. W. Thrush, comp. and ed., *A Dictionary of Mining, Mineral, and Related Terms,* Bureau of Mines, 1968; *A DOD Glossary of Mapping, Charting and Geodetic Terms,* Department of Defense, 1967; J. M. Gilliland, *Solar-Terrestrial Physics: A Glossary of Terms and Abbreviations,* Royal Aircraft Establishment Technical Report 67158, 1967; W. H. Allen, ed., *Dictionary of Technical Terms for Aerospace Use,* National Aeronautics and Space Administration, 1965; *Glossary of Stinfo Terminology,* Office of Aerospace Research, U.S. Air Force, 1963; *Naval Dictionary of Electronic, Technical, and Imperative Terms,* Bureau of Naval Personnel, 1962; R. E. Huschke, *Glossary of Meteorology,* American Meteorological Society, 1959; *ADP Glossary,* Department of the Navy, NAVSO P-3097; *Glossary of Air Traffic Control Terms,* Federal Aviation Agency; *A Glossary of Range Terminology,* White Sands Missile Range, New Mexico, National Bureau of Standards, AD 467-424; *Nuclear Terms: A Glossary,* 2d ed., Atomic Energy Commission.

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veronal [PHARM] A white crystalline powder formerly used medicinally as a hypnotic drug to induce sleep. Also known as barbital. { 'ver·ə,nȯl }

verrou *See* riegel. { və'rü }

verruca [BIOL] A wartlike elevation on the surface of a plant or animal. { və'rü·kə }

verruca peruana *See* verruca peruviana. { və'rü·kə ,per·ə'wä·nə }

verruca peruviana [MED] A benign eruptive form of bartonellosis with chronic cutaneous lesions. Also known as verruca peruana. { və'rü·kə pə,rü·vē'a·nə }

Verrucariaceae [BOT] A family of crustose lichens in the order Pyrenulales typically found on rocks, especially in intertidal or salt-spray zones along rocky coastlines. { və,rü·kə·rē'ās·ē,ē }

Verrucomorpha [INV ZOO] A suborder of the crustacean order Thoracica composed of sessile, asymmetrical barnacles. { və,rü·kə'mȯr·fə }

verrucose [BIOL] Having the surface covered with wartlike protuberances. { və'rü,kōs }

verrucous endocarditis [MED] Small thrombotic, nonbacterial, wartlike lesions on the heart valves and endocardium, occurring frequently in systemic lupus erythematosus. Also known as terminal endocarditis. { və'rü·kəs ¦en·dō·kär'dīd·əs }

vers *See* versed sine.

versatile anther [BOT] An anther whose attachment is near its middle, thus enabling it to swing freely. { 'vər·səd·əl 'an·thər }

versatile automatic test equipment [ELECTR] Computer-controlled tester, for missile electronic systems, that troubleshoots faults by deductive logic and isolates them to the plug-in module or component level. { 'vər·səd·əl ¦ȯd·ə¦mad·ik 'test i,kwip·mənt }

versed cosine *See* coversed sine. { 'vərst 'kō,sīn }

versed sine [MATH] The versed sine of A is $1 -$ cosine A. Denoted vers. Also known as versine. { 'vərst 'sīn }

versiera *See* witch of Agnesi. { ,vər·sē'er·ə }

versine *See* versed sine. { 'vər,sīn }

verso [GRAPHICS] **1.** A left-hand page in a book, usually carrying an even page number. **2.** The side of a page that is to be read second. { 'vər·sō }

vertebra [ANAT] One of the bones that make up the spine in vertebrates. { 'vərd·ə·brə }

vertebral arch [ANAT] An arch formed by the paired pedicles and laminas of a vertebra; the posterior part of a vertebra which together with the anterior part, the body, encloses the vertebral foramen in which the spinal cord is lodged in vertebrates. Also known as neural arch. { 'vərd·ə·brəl 'ärch }

vertebral column *See* spine. { 'vərd·ə·brəl 'käl·əm }

Vertebrata [VERT ZOO] The major subphylum of the phylum Chordata including all animals with backbones, from fish to human. { ,vərd·ə'bräd·ə }

vertebrate zoology [ZOO] That branch of zoology concerned with the study of members of the Vertebrata. { 'vərd·ə·brət zō'äl·ə·jē }

vertebratus [METEOROL] A cloud variety (applied mainly to the genus cirrus), the elements of which are arranged in a manner suggestive of vertebrae, ribs, or a fish skeleton. { ,vərd·ə'bräd·əs }

vertex [ASTRON] **1.** The highest point that a celestial body attains. **2.** On a great circle, that point that is closest to a pole. [MATH] **1.** For a polygon or polyhedron, any of those finitely many points which together with line segments or plane pieces determine the figure or solid. **2.** The common point at which the two sides of an angle intersect. **3.** The fixed point through which pass all the elements of a cone or conical surface. **4.** An intersection of a conic with one of its axes of symmetry. **5.** A member of the set of points that are connected by the edges. Also known as node. **6.** For a simplex, one of the finite set of points on which a simplex is based. [OPTICS] One of the points where the surface of a lens intersects the optical axis. { 'vər,teks }

vertex angle [MATH] In a triangle, the angle opposite the base. { 'vər,teks ,aŋ·gəl }

vertex cover [MATH] A set of vertices in a graph such that every edge in the graph is incident to at least one vertex in this set. { 'vər,teks ¦kəv·ər }

vertex-covering number [MATH] For a graph, the smallest possible number of vertices in a vertex cover. { 'vər,teks ¦kəv·ər·iŋ ,nəm·bər }

vertex detector [NUCLEO] A particle detector designed to provide high-precision (typically 5 to 20 micrometers) measurements of points along the trajectories of charged particles very close to the interaction point (typically 0.4 to 4 inches, or 1 to 10 centimeters) of a high-energy collider. { 'vər,teks di,tek·tər }

vertex-disjoint paths [MATH] In a graph, two paths with common end points that have no other points in common. { ¦vər,teks 'dis,jȯint ,paths }

vertex domination number [MATH] For a graph, the smallest possible number of vertices in a dominating vertex set. Also known as external stability number. { ¦vər,teks ,däm·ə'nā·shən ,nəm·bər }

vertex-induced graph [MATH] A subgraph whose edges consist of all the edges in the original graph that join pairs of vertices in the subgraph. Also known as induced subgraph. { ¦vər,teks in,düst 'graf }

vertex power [OPTICS] The reciprocal of the back focal length of a lens. { 'vər,teks ,pau̇·ər }

vertical air photograph [GRAPHICS] An air photograph taken with the optical axis of the camera perpendicular to the earth's surface. { 'vərd·ə·kəl 'er 'fōd·ə,graf }

vertical anemometer [METEOROL] An instrument which records the vertical component of the wind speed. { 'vərd·ə·kəl ,an·ə'mäm·əd·ər }

vertical angles [MATH] The two angles produced by a pair of intersecting lines and lying on opposite sides of the point of intersection. { 'vərd·ə·kəl 'aŋ·gəlz }

vertical antenna [ELECTROMAG] A vertical metal tower, rod, or suspended wire used as an antenna. { 'vərd·ə·kəl an'ten·ə }

vertical axis [NAV ARCH] The vertical line near the center of gravity of a craft, perpendicular to both the longitudinal and lateral axes, around which it yaws. { 'vərd·ə·kəl 'ak·səs }

vertical-axis propeller [NAV ARCH] A type of propeller wheel that consists of a circular horizontal disk set flush into a vessel's bottom, rotates about a vertical axis, and carries near its periphery a number of spadelike vertical blades. { 'vərd·ə·kəl ¦ak·səs prə'pel·ər }

vertical ballistic transistor [ELECTR] A transistor in which, ideally, electrons traverse ballistically (that is without scattering) a very short region that separates a cathode from an anode, and whose effective cross section is modulated by the potential of metal gate contacts. { ¦vərd·i·kəl bə,lis·tik tran'zis·tər }

vertical band saw [MECH ENG] A band saw whose blade operates in the vertical plane; ideal for contour cutting. { 'vərd·ə·kəl 'band ,sȯ }

vertical blanking [ELECTR] Blanking of a television picture tube during the vertical retrace. { 'vərd·ə·kəl 'blaŋk·iŋ }

vertical boiler [MECH ENG] A fire-tube boiler having vertical tubes between top head and tube sheet, connected to the top of an internal furnace. { 'vərd·ə·kəl 'bȯi·lər }

vertical boring mill [MECH ENG] A large type of boring machine in which a rotating workpiece is fastened to a horizontal table, which resembles a four-jaw independent chuck with extra radial T slots, and the tool has a traverse motion. { 'vərd·ə·kəl 'bȯr·iŋ ,mil }

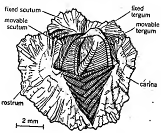
vertical broaching machine [MECH ENG] A broaching machine having the broach mounted in the vertical plane. { 'vərd·ə·kəl 'brōch·iŋ mə,shēn }

vertical-cavity surface-emitting laser [OPTICS] A very small semiconductor laser in which stacks of dielectric mirrors above and below the optically active region form the optical cavity, the active gain medium consists of one or more semiconductor quantum wells placed parallel to the mirrors at an antinode of the cavity resonance, and lasing light emission is from the surface of the semiconductor substrate, normal to the plane of the gain medium. Abbreviated VCSEL. { ¦vərd·i·kəl ,kav·əd·ē ¦sər·fəs i,mid·iŋ 'lā·zər }

vertical centering control [ELECTR] The centering control provided in a television receiver or cathode-ray oscilloscope to shift the position of the entire image vertically in either direction on the screen. { 'vərd·ə·kəl 'sen·tər·iŋ kən¦trōl }

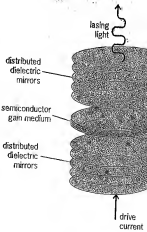
vertical center keelson [NAV ARCH] The lower middle line girder, which, in conjunction with a flat plate keel on the bottom and a rider plate on top, forms the principal fore-and-aft strength member in the bottom of a ship. { 'vərd·ə·kəl 'sen·tər 'kēl·sən }

VERRUCOMORPHA



Verruca stroemia.

VERTICAL-CAVITY SURFACE-EMITTING LASER



Distributed dielectric mirrors form the optical cavity. Lasing light is normal to the plane of the gain medium.

DISCRETE MATHEMATICS

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Richard Johnsonbaugh

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Well, I got on the road, and I went north to Providence. Met the Mayor.
The Mayor of Providence!
He was sitting in the hotel lobby.
What'd he say?
He said, "Morning!" And I said, "You got a fine city here, Mayor."
And then he had coffee with me.
And then I went to Waterbury.
Waterbury is a fine city. Big clock city, the famous Waterbury clock.
Sold a nice bill there. And then Boston—Boston is the cradle of the Revolution. A fine city. And a couple of other towns in Mass., and on to Portland and Bangor and straight home!

—from Death of a Salesman

6

GRAPH THEORY

Although the first paper in graph theory goes back to 1736 (see Example 6.2.16) and several important results in graph theory were obtained in the nineteenth century, it is only since the 1920s that there has been a sustained, widespread, intense interest in graph theory. Indeed, the first text on graph theory ([König]) appeared in 1936. Undoubtedly, one of the reasons for the recent interest in graph theory is its applicability in many diverse fields, including computer science, chemistry, operations research, electrical engineering, linguistics, and economics.

We begin with some basic graph terminology and examples. We then discuss some important concepts in graph theory, including paths and cycles. A shortest-path algorithm is presented that efficiently finds a shortest path between two given points. Two classical graph problems, the existence of Hamiltonian cycles and the traveling salesperson

† This section can be omitted without loss of continuity.

problem, are then considered. After presenting ways of representing graphs, we study the question of when two graphs are essentially the same (i.e., when two graphs are isomorphic) and when a graph can be drawn in the plane without having any of its edges cross. We conclude by presenting a solution based on a graph model to the Instant Insanity puzzle.

6.1 INTRODUCTION

Figure 6.1.1 shows the highway system in Wyoming that a particular person is responsible for inspecting. Specifically, this road inspector must travel all of these roads and file reports on road conditions, visibility of lines on the roads, status of traffic signs, and so on. Since the road inspector lives in Greybull, the most economical way to inspect all of the roads would be to start in Greybull, travel each of the roads exactly once, and return to Greybull. Is this possible? See if you can decide before reading on.

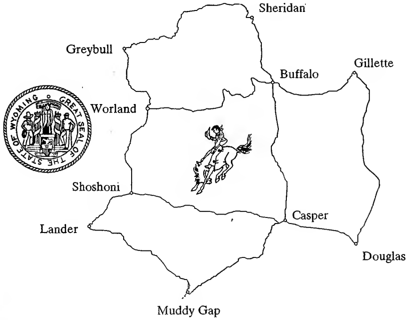

FIGURE 6.1.1 Part of the Wyoming highway system.

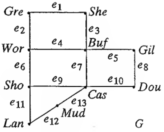

FIGURE 6.1.2
A graph model of the highway system shown in Figure 6.1.1.

The problem can be modeled as a **graph**. In fact, since graphs are drawn with dots and lines, they look like road maps. In Figure 6.1.2, we have drawn a graph G that models the map of Figure 6.1.1. The dots in Figure 6.1.2 are called **vertices** and the lines that connect the vertices are called **edges**. (Later in this section we will define all of these terms carefully.) We have labeled each vertex with the first three letters of the city to which it corresponds. We have labeled the edges $e_1, \ldots, e_{13}$. In drawing a graph, the only information of importance

is which vertices are connected by which edges. For this reason, the graph of Figure 6.1.2 could just as well be drawn as in Figure 6.1.3.

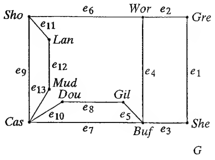

FIGURE 6.1.3 An alternative, but equivalent, graph model of the highway system shown in Figure 6.1.1.

If we start at a vertex v_0, travel along an edge to vertex v_1, travel along another edge to vertex v_2, and so on, and eventually arrive at vertex v_n, we call the complete tour a **path** from v_0 to v_n. The path that starts at *She*, then goes to *Buf*, and ends at *Gil* corresponds to a trip on the map of Figure 6.1.1 that begins in Sheridan, goes to Buffalo, and ends at Gillette. The road inspector's problem can be rephrased for the graph model G in the following way: Is there a path from vertex *Gre* to vertex *Gre* that traverses every edge exactly once?

We can show that the road inspector cannot start in Greybull, travel each of the roads exactly once, and return to Greybull. To put the answer in graph terms, there is no path from vertex *Gre* to vertex *Gre* in Figure 6.1.2 that traverses every edge exactly once. To see this, suppose that there is such a path and consider vertex *Wor*. Each time we arrive at *Wor* on some edge, we must leave *Wor* on a different edge. Furthermore, every edge that touches *Wor* must be used. Thus the edges at *Wor* occur in pairs. It follows that an even number of edges must touch *Wor*. Since three edges touch *Wor*, we have a contradiction. Therefore, there is no path from vertex *Gre* to vertex *Gre* in Figure 6.1.2 that traverses every edge exactly once. The argument applies to an arbitrary graph G. If G has a path from vertex v to v that traverses every edge exactly once, an even number of edges must touch each vertex. We discuss this problem in greater detail in Section 6.2.

At this point we give some formal definitions.

DEFINITION 6.1.1

A *graph* (or *undirected graph*) G consists of a set V of *vertices* (or *nodes*) and a set E of *edges* (or *arcs*) such that each edge $e \in E$ is associated with an unordered pair of vertices. If there is a unique edge e associated with the vertices v and w, we write $e = (v, w)$ or $e = (w, v)$. In this context, (v, w) denotes an edge between v and w in an undirected graph and *not* an ordered pair.